647510 **BSCalgCC5010** **Seat No :________**

B.Sc. Semester - 5 (*CBCS*) Examination
Oct/Nov. - 2021(Old Course)
MATHEMATICAL ANALYSIS-1 AND ABSTRACT ALGEBRA-1 (CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Question-1 (A) Answer any one out of two. **[7]**

(1) X is metric space and $E \subset$ X. Prove that $\overline{E}$ is closed.

(2) Explain cantor set and define it. Show that $\frac{1}{4} \in$ C.

(B) Answer any one out of two. **[4]**

(1) d is define on non empty set X as; for $\forall$ x,y $\in$ X

$$d(x,y) = \begin{cases} 0 & ; \ x=y \\ 1 & ; \ x \neq y \end{cases}$$

Then show that (X,d) is metric space.

(2) X is metric space. Show that union of finite numbers of closed subsets of X is closed. Explain with example that this statement is not true for infinite numbers of closed subsets of X.

(C) Answer any one out of two. **[3]**

(1) (i) R is discrete metric space. Find N(3,0.5) in R.

(ii) Check whether set E=$(-\infty,5) \cup \{10\}$is open and closed.

(2) in usual notation show that $(A \cap B)^0 = A^0 \cap B^0$.

Question-2 (A) Answer any one out of two. **[7]**

(1) Prove that any bounded function f defined on [a,b] is R-integrable iff For $\forall$ $\varepsilon>0$ there exist partition P of [a,b] such that U(P,f) - L(P,f) < ε , Where $\|P\| < \delta$.

(2) If functions f and g are R-integrable in [a,b] then show that f+g is also R-integrable in [a,b] and

$$\int_a^b [f(x)+g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$$

(B) Answer any one out of two. **[4]**

(1) Find $\int_0^1 x^2$ dx using definition of R-integration.

(2) Function f is bounded in [a,b]. P and P^* be two partitions of [a,b] Such that $P \subset P^*$ Then show that $U(P^*,f) \leq U(P,f)$

(C) Answer any one out of two. **[3]**

(1) Using darbour's theorem find value of

$$\lim_{n\to\infty} \frac{1}{n}\left[\left(\frac{1}{n}\right)^2 + \left(\frac{2}{n}\right)^2 + ...+ \left(\frac{n}{n}\right)^2\right]$$

(2) If bounded function f on [a,b] is continuous then show that f is R-integrable.

Question-3 (A) Answer any one out of two. **[7]**

(1) If bounded function f is R-integrable in [a,b] and F(x)= $\int_a^x f(t)dt$ Where $a \leq x \leq b$ then show that F(x) is continuous in [a,b].

(2) G=R-{1}. Binary operation * is defined on G as ; for a,b $\in$ G , a*b=a+b-ab then show that (G,*) is group and find 3^{-1} in G

(B) Answer any one out of two. **[4]**

(1) State and prove general form of first mean value theorem for R-integration.

(2) Define: Group , Binary operation. Show that equation a*x=b has unique solution in Group (G,*). Where a ,b $\in$ G.

(C) Answer any one out of two. **[3]**

(1) Show that $\frac{\pi^2}{6} \leq \int_0^{\pi} \frac{x}{2+\cos(x)} dx \leq \frac{\pi^2}{2}$

(2) Show that G= $\{\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} / \theta \epsilon R\}$ is group under matrix Multiplication.

Question-4 (A) Answer any one out of two. **[7]**

(1) State and prove lagrange's theorem.

(2) Define: Symmetric group. Find number of elements of Symmetric group S_n. Also write all elements of S_3.

(B) Answer any one out of two. **[4]**

(1) Prove that $H=\{x \in G/xa=ax\}$ is subgroup of group G. where a is invariant element of G.

(2) Define alternating group and prepare group table for alternating group A_3 in usual notation.

(C) Answer any one out of two. **[3]**

(1) $\sigma \epsilon$ S_6 where $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix}$ then find σ^{-1} and o(σ). Also Check whether σ is even or odd.

(2) Prove that group and its subgroup has same identity element. Also prove that inverse of any element in group and its subgroup is also same.

Question-5 (A) Answer any one out of two. **[7]**

(1) State and prove Cayley's theorem.

(2) G is finite group and a ϵ G. prove that O(a)/O(G).

(B) Answer any one out of two. **[4]**

(1) Prove that every permutation f of finite set is a product of disjoint cycles.

(2) For n≥2, in usual notation show that A_n is normal subgroup of S_n.

(C) Answer any one out of two. **[3]**

(1) Prepare group table in usual notation for quotient group S_3/A_3.

(2) If $(G,*) \cong (G',*')$. Show that G is commutative iff G' is commutative. Where G and G' are groups.

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